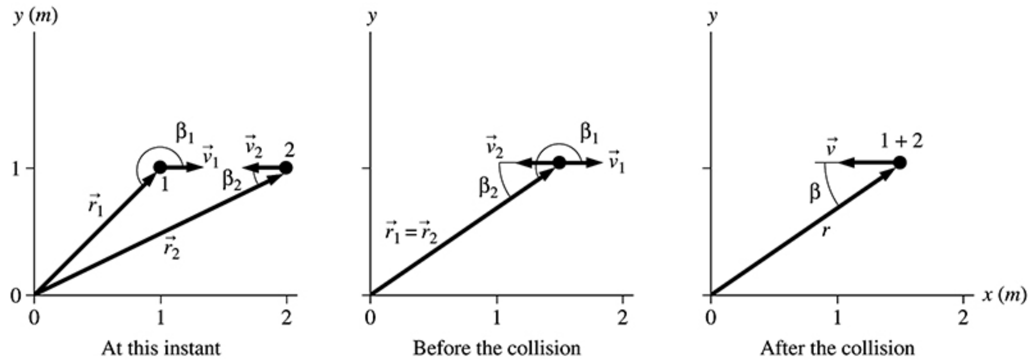
12.82. Model: The clay balls are particles and undergo a totally inelastic collision. Linear momentum is conserved during the collision.

Visualize:



Solve: (a) The angle is measured counterclockwise from $\vec{r}$ to $\vec{v}$. From geometry, $\beta_1 = 180° + \tan^{-1}(1) = 225°$, and $\beta_2 = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$. So

$$\begin{aligned} L &= L_1 + L_2 + m_1 r_1 v_1 \sin\beta_1 + m_2 r_2 v_2 \sin\beta_2 \\ &= (0.015\text{ kg})(\sqrt{2}\text{ m})(2\text{ m/s})\sin 225° + (0.025\text{ kg})(\sqrt{5}\text{ m})(2.0\text{ m/s})\sin 26.6° \\ &= 0.020\text{ kg m}^2\text{/s}. \end{aligned}$$

Note that the signs of L_1 and L_2 agree with those determined by the right-hand rule.

(b) At the instant before the clay balls collide they are located at (1.5 m, 1.0 m). Here,

$$\beta_2 = \tan^{-1}\left(\frac{2}{3}\right) = 33.7° \qquad \beta_1 = 180° + \beta_2 = 146.3°$$

So

$$\begin{aligned} L &= (0.015\text{ kg})\left(\sqrt{(1.0\text{ m})^2 + (1.5\text{ m})^2}\right)(2.0\text{ m/s})\sin 213.7 \\ &+ (0.025\text{ kg})\left(\sqrt{(1.0\text{ m})^2 + (1.5\text{ m})^2}\right)(2.0\text{ m/s})\sin 33.7° \\ &= 0.020\text{ kg m}^2\text{/s} \end{aligned}$$

(c) The clay balls have a final speed v after the collision. Linear momentum is conserved.

$$\begin{aligned} & p_{1i} + p_{2i} = p_{(1+2)f} \\ & (0.015\text{ kg})(2.0\text{ m/s}) + (0.025\text{ kg})(-2.0\text{ m/s}) = (0.015\text{ kg} + 0.025\text{ kg})v \\ & \Rightarrow v = -0.50\text{ m/s}. \end{aligned}$$

The balls are moving to the left.

The angle $\beta = \beta_2$ from part (b). The angular momentum after the collision is

$$L = (0.040\text{ kg})\left(\sqrt{(1.0\text{ m})^2 + (1.5\text{ m})^2}\right)(0.50\text{ m/s})\sin 33.7° = 0.020\text{ kg m}^2\text{/s}$$

Assess: Angular momentum is also conserved since $\vec{\tau}_{\text{net}} = 0$.